

Q8. If $f(x) = x^2 - 4x + 6$ is increasing in the interval:

- A (0,2) B $(-\infty, 2]$ C $[2, \infty)$ D $[1, 2]$

Q9. A cylindrical tank of radius 10cm is being filled with sugar at the rate of 100π c. c per sec. The rate, at which the height of the sugar inside the tank is increasing is:

- A 0.1 cm/s B 0.5 cm/s C 1cm/s D 1.1 cm/s

Q10. The values of k so that $f(x) = \sin x - \cos x - kx + 5$ decreases for all real values of x are:

- A $1 < k < \sqrt{2}$ B $k \geq 1$ C $k \geq \sqrt{2}$ D $k < 1$

Q11. If $f(x) = 2x + \cos x$, then $f(x)$:

- A is an increasing function B has a minimum at $x = \pi$ C has a maximum at $x = \pi$ D is a decreasing function

Q12. The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at :

- A (1, -10) B (1, 10) C (10, 1) D (-10, 1)

Q13. A spherical ball has a variable diameter $\frac{5}{2}(3x + 1)$. The rate of change of its volume with respect to x , when $x = 1$:

- A 225π B 300π C 375π D 125π

Q14. Absolute minimum value of $f(x) = (x - 2)^2 + 5$ in the interval $[-3, 2]$:

- A -3 B 2 C 5 D 30

Q15. (A): $f(x) = x^x$ has a local maxima at $x = e$.

(R): $f(x)$ has a local maximum at $x = a$, if $f'(a) = 0$ and $f''(a) < 0$

- A A and (R) are true and R is the correct explanation of (A) B A and (R) are true and R is not the correct explanation of (A)
C A is true but (R) is false D A is false but (R) is true

Section B- Short answer type (2 marks)

Q16. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

Q17. Find the interval/s in which the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = xe^x$, is increasing.

- Q18. If $f(x) = x + \frac{1}{x}$, $x \geq 1$, show that f is an increasing function.
- Q19. Amongst of all pairs of positive integers, with product is 289, find which of the two numbers add up to the least.
- Q20. Find the local maxima and local minima, if any of the function f , given by $f(x) = \sin x + \cos x$, $0 < x < \frac{\pi}{2}$
- Q21. Show that the function $f(x) = (x^3 - 6x^2 + 12x + 18)$ is an increasing function
- Q22. Find the maximum profit that a company can make, if the profit function is given by $P(x) = 72 + 42x - x^2$, where x is the number of units and P is the profit in rupees.
- Q23. Sand is pouring from a pipe at the rate of $12 \text{ cm}^3/\text{s}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?
- Q24. Show that the surface area of a closed cuboid with a square base and given volume is minimum, when it is a cube.
- Q25. The volume of a wooden block in the shape of a cube increases at a constant rate as the air becomes moist during the rainy season. Show that the rate of change of its surface area varies inversely as the length of edge of the cube.

Case Study Based Questions

- Q26. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark. A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ . On the basis of the above information, answer the following questions :
- Express θ in terms of height of the camera installed on the pole and x .
 - Find $\frac{d\theta}{dx}$.
 - (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. OR
(a) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car.

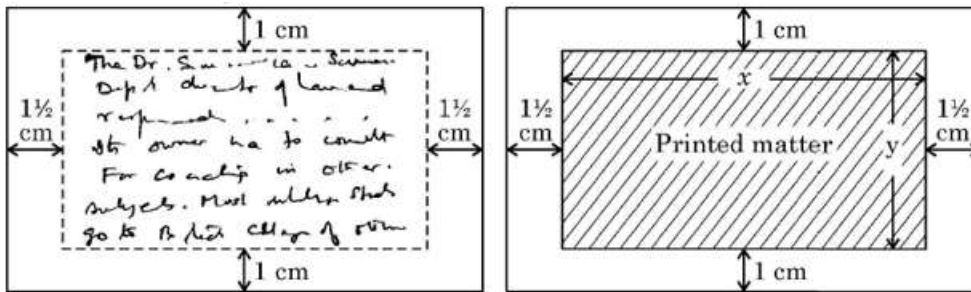
Q27. A tank, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank. A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2 \text{ cm}^3/\text{s}$. The semi-vertical angle of the conical tank is 45° .

On the basis of given information, answer the following questions :

- (i) Find the volume of water in the tank in terms of its radius r .
- (ii) Find rate of change of radius at an instant when $r = 2\sqrt{2} \text{ cm}$.
- (iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2} \text{ cm}$. OR

(b) Find the rate of change of height h at an instant when height is 4 cm .

Q28. A rectangular visiting card is to contain 24 sq.cm of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right be 1.5 cm as shown below.



On the basis of this information answer the following:

- i) Write the area of the visiting card in terms of x .
- ii) Obtain the dimensions of the visiting card for minimum area.

Q29. A company produces cylindrical tumblers open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.

Based on the above information answer the following:

If for a tumbler, V is its volume, h be the height and r the radius of the circular base,

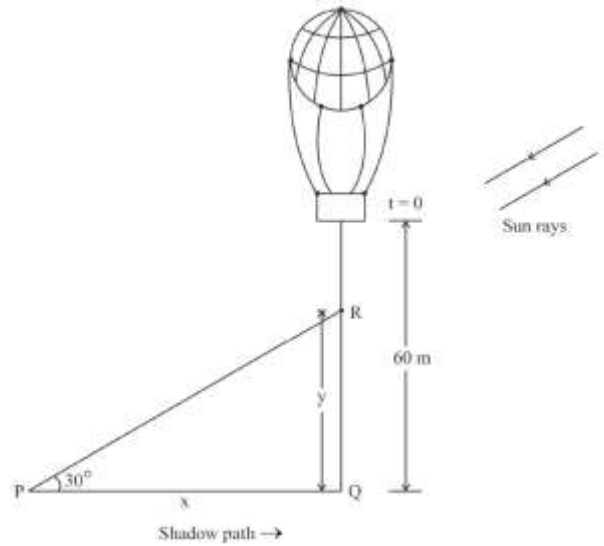
- (i) Differentiate its volume with respect to radius of the base, where surface area is constant.
- (ii) If the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base.

Q30. A sandbag is dropped from a balloon at a height of 60 metres.

When the angle of elevation of the sun is 30° , the position of the sandbag is given

by the equation $y = 60 - 4.9t^2$, where y is the height of the sandbag above the ground and t is the time in seconds.

On the basis of the above information, answer the following questions:



- (i) Find the relation between x and y , where x is the distance of the shadow at P from the point Q and y is the height of the sandbag above the ground.
- (ii) After how much time will the sandbag be 35 metres above the ground?
- (iii) (a) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

OR

- (iii) (b) How fast is the height of the sandbag decreasing when 2 seconds have elapsed?

31. The equation of the path traced by a roller-coaster is given by the polynomial

$f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following:

- i) Find the value of a .
- ii) Find second derivative of $f(x)$ at $x = 1$.

ANSWER KEY

Q1.	B	Q2	A	Q3	D	Q4	A
Q5.	A	Q6	A	Q7	C	Q8	C
Q9.	C	Q10	C	Q11	A	Q12	A
Q13.	C	Q14	C	Q15	D	Q17	$[-1, \infty)$
Q19.	17, 17	Q20.	Local max at $x = \frac{\pi}{4}$, and local maximum value $= \sqrt{2}$	Q22	Maximum profit at $x = 21$ and maximum profit ₹ 513.	Q23	$\frac{1}{48\pi}$ cm/s
Q26	<i>i)</i> $\theta = \tan^{-1} \left(\frac{5}{x} \right)$ <i>ii)</i> $\frac{d\theta}{dx} = \frac{-5}{x^2+25}$ <i>iii)</i> a) $-\frac{4}{101}$ rad/s OR Or b) 15 m/s			Q27.	<i>i)</i> $V = \frac{1}{3} \pi r^3$ <i>ii)</i> $\frac{-1}{4\pi}$ cm/s <i>iii)</i> a) $-2\text{cm}^2/\text{s}$ OR b) $\frac{-1}{8\pi}$ cm/s		
Q28	ii) The dimension of the card with minimum area is, length = 9 cm, breadth = 6 cm			Q29.	Maximum volume when height = radius		
Q30.	<p>(i) $\frac{y}{x} = \tan 30^\circ \Rightarrow y = \frac{x}{\sqrt{3}}$ or $x = \sqrt{3}y$</p> <p>(ii) Using $y = 35\text{m} \Rightarrow 60 - 4.9t^2 = 35 \Rightarrow 4.9t^2 = 25 \Rightarrow t = \frac{5\sqrt{10}}{7}$ seconds.</p> <p>(iii) (a) $x = \sqrt{3}y \Rightarrow x = 60\sqrt{3} - 4.9\sqrt{3}t^2 \Rightarrow \left. \frac{dx}{dt} \right]_{t=\frac{5\sqrt{10}}{7}} = -4.9\sqrt{3}(2t) \Big]_{t=\frac{5\sqrt{10}}{7}} = -7\sqrt{30}$ m/s</p>						
Q31	<p>i) $a = \frac{1}{27}$</p> <p>ii) $f'(x) = \frac{1}{27} (3x^2 + 14x - 21)$ $f''(x) = \frac{6x + 14}{27}$ $f''(1) = \frac{20}{27}$</p>						
